University of Diyala
College of Engineering
Department of Materials

Fundamentals of Electric Circuits

Lecture Five

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Basic Laws

2-6 Ohm's Law

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$\boldsymbol{v \propto i}$$

$$\boldsymbol{\Rightarrow v = iR}$$

We may deduce from Eq. that

$$R = \frac{v}{i}$$

Example 1/ A voltage source of $20\,sin\,\pi t$ V is connected across a $5\ K\Omega$ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20\sin\pi t}{5\times 10^3} = 4\sin\pi t\ \text{mA}$$

Hence,

$$p = vi = 80\sin^2\pi t\ \text{mW}$$

H.W. / A resistor absorbs an instantaneous power of $20cos^2t\ mW$ when connected to a voltage source $v = 10\ cos\ t\ V$. Find i and R.

Answer $2\ cos\ t\ mA$, $5K\Omega$

A useful quantity in circuit analysis is the reciprocal of resistance R, known as *conductance* and denoted by *G*:

$$\boldsymbol{G = \frac{1}{R} = \frac{i}{v}} \qquad (\boldsymbol{i})$$

Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).

The same resistance can be expressed in ohms or siemens. For example, 10 Ω is the same as 0.1 S. From Eq. (i), we may write

$$i = Gv$$

The power dissipated by a resistor can be expressed in terms of R.

$$p = vi = i^2 R = \frac{v^2}{R}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$p = vi = v^2 G = \frac{i^2}{G}$$

Example 2

In the circuit shown in Fig. 1, calculate the current i, the conductance G, and the power p.

Fig. 1

2-7 Nodes, Branches, and Loops

A branch represents a single element such as a voltage source or a resistor.

In other words, a branch represents any two-terminal element. The circuit in Fig. 2 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

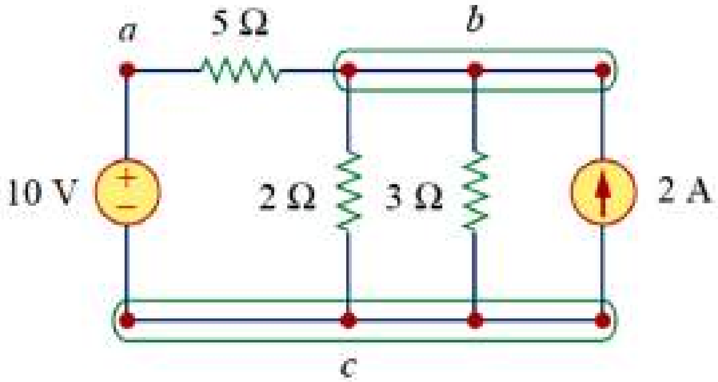


Fig 2

A node is the point of connection between two or more branches.

We demonstrate that the circuit in Fig. 3 has only three nodes by redrawing the circuit in Fig. 3.

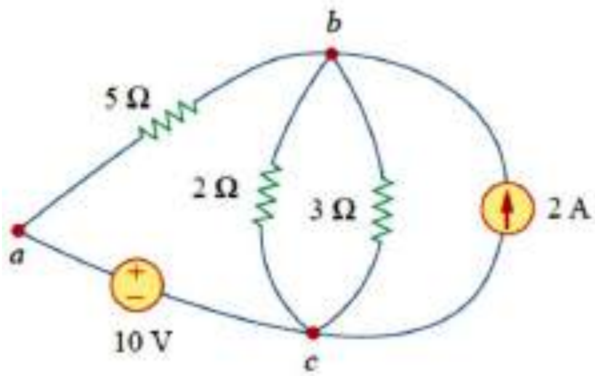


Fig. 3. The three-node circuit of Fig. 2.1 is redrawn.

A loop is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1$$

Example 3

How many branches and node does the circuit in Fig. 4 have?

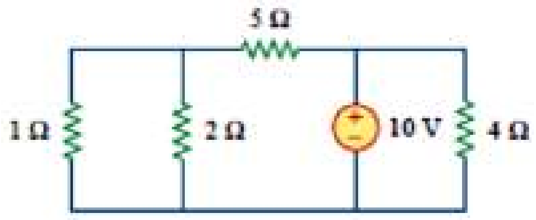


Fig. 4

2-8 Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's current law (KCL) *states that the algebraic sum of currents entering a node (or a closed boundary) is zero.*

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0 \qquad (1)$$

Where N is the number of branches connected to the node and i_n is the *nth* current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

In Fig. 5, for example, the shaded area can enclose an entire system or a complex network, or it can simply provide a connection point (junction) for the displayed currents. In each case, the current entering must equal that leaving, as required by above Eq.

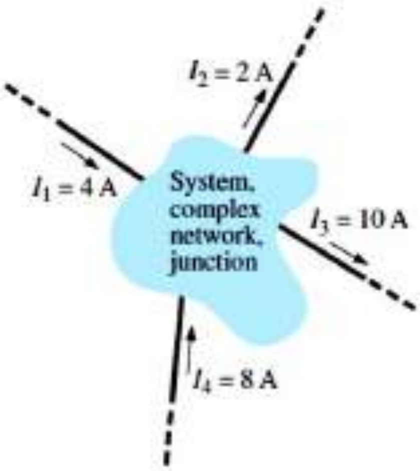


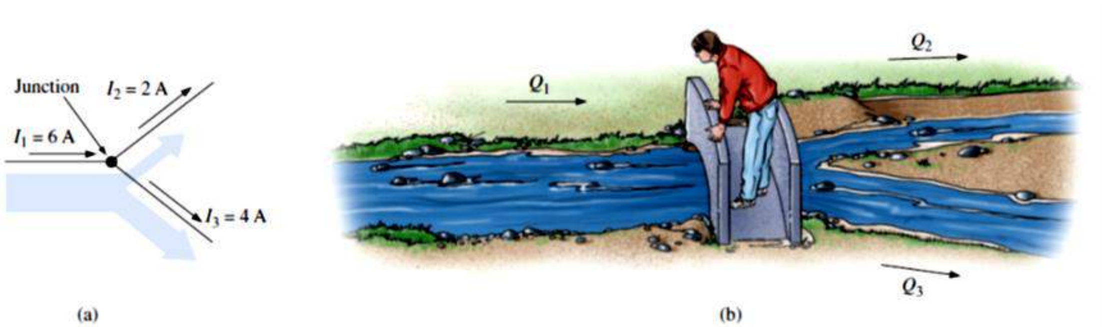
Fig. 5 *Introducing Kirchhoff 's current law.*

$$\sum I_i = \sum I_o$$

$$I_1 + I_4 = I_2 + I_3$$

$$4\,A + 8\,A = 2\,A + 10\,A$$

$$12\,A = 12\,A$$



The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Example 4/ Determine currents I_3 and I_4 in Fig. 6 using Kirchhoff's current law.

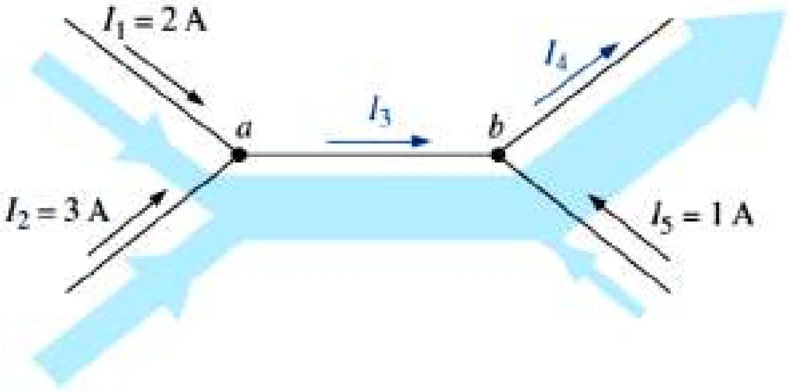


Fig. 6 *Two-node configuration for Example.*

Example 5- Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 7.

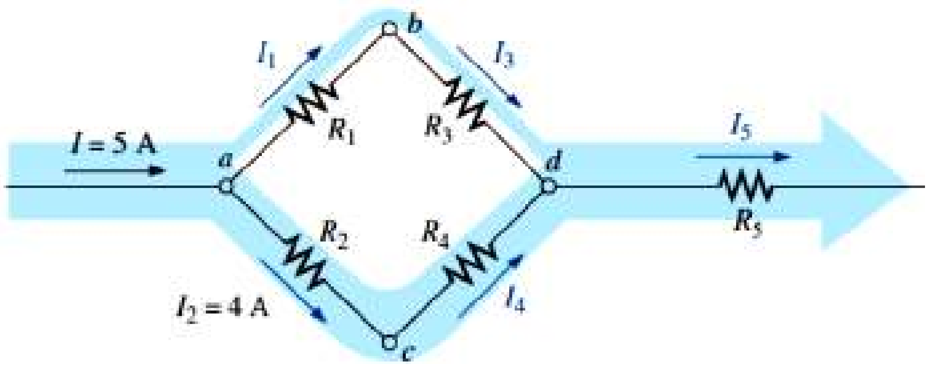


Fig. 7 *Four-node configuration for Example*

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0 \qquad (2)$$

Where M is the number of voltages in the loop (or the number of branches in the loop) and V_m is the mth voltage.

To illustrate KVL, consider the circuit in Fig. 8. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1, +v_2, +v_3, -v_4$ and $+v_5$ in that order.

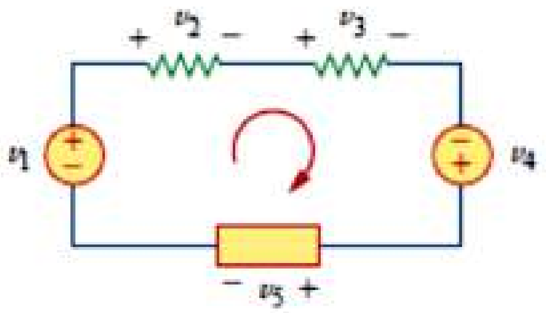


Fig. 8

For example, as we reach branch 3, the positive terminal is met first; hence, we have, $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$ Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4$$

Which may be interpreted as

Sum of voltage drops = Sum of voltage rises

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources. For example, for the voltage sources shown in Fig. 9(a), the combined or equivalent voltage source in Fig. 9(b) is obtained by applying KVL.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

Or

$$V_{ab} = V_1 + V_2 - V_3$$

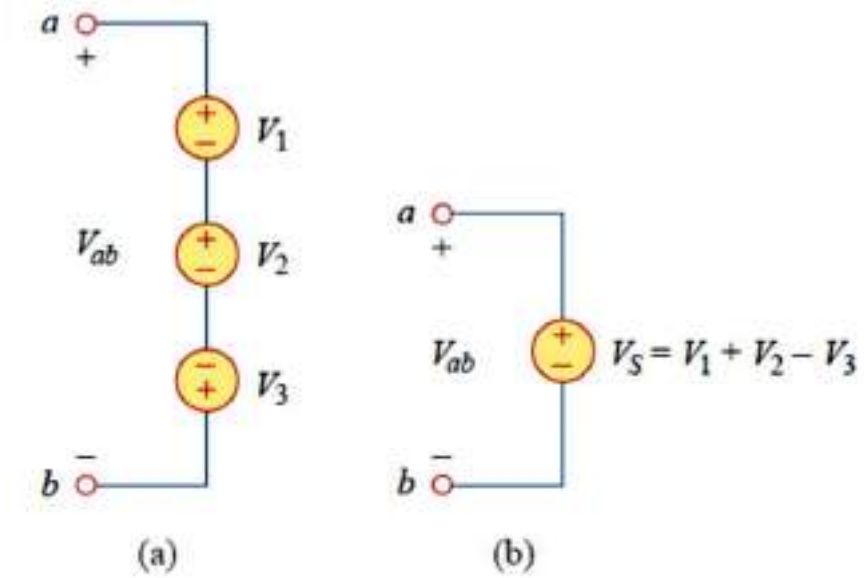
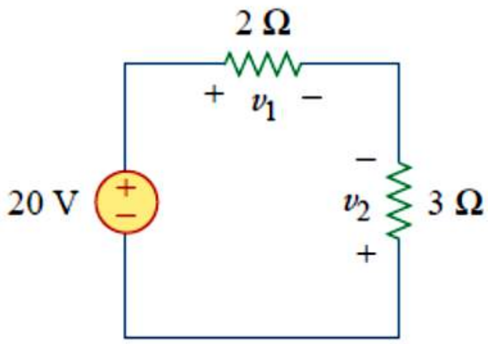


Fig. 9 Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Example 6: For the circuit in Fig. 10(a), find voltages v_1 and v_2.



(a)

Fig. 10

Example 7: For the circuit in Fig. 10, find voltages v_1 and v_2.

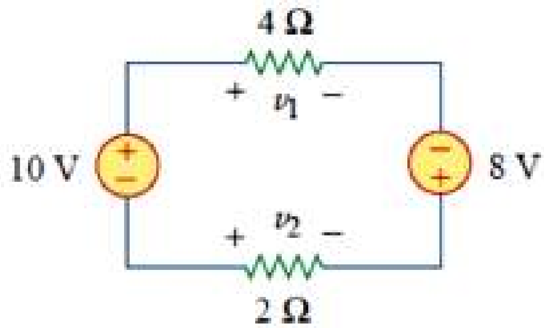


Fig. 11

Fig. 13

Example 8: Find current i_o and voltage v_o in the circuit shown in Fig. 12.

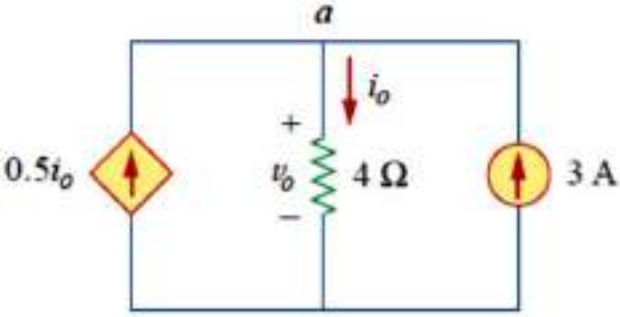


Fig. 12

Example 9: Find current i_o and voltage v_o in the circuit shown in Fig. 13.

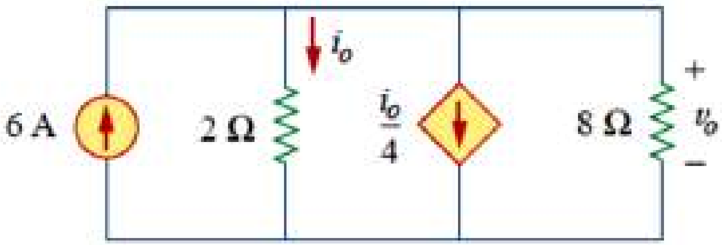


Fig. 13

Example 10: Find the currents and voltages in the circuit shown in Fig. 14 (a)

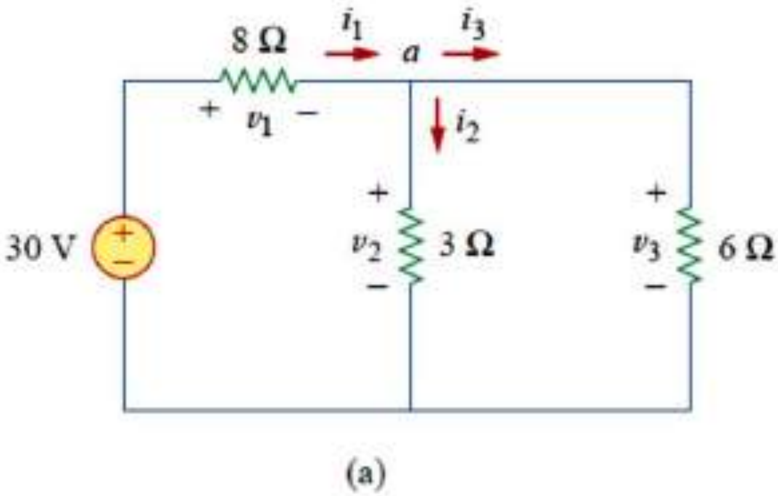
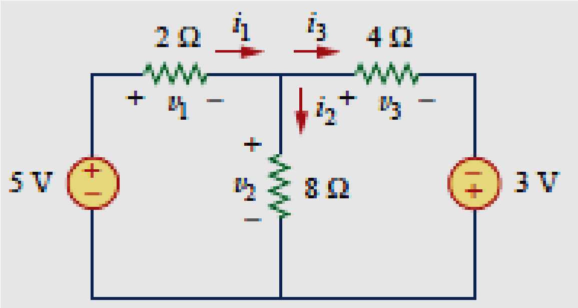


Fig. 14

H.W. Find the currents and voltages in the circuit shown in Fig. 15



$$v_1 = 3\ V, v_2 = 2\ V, v_3 = 5\ V, i_1 = 1.5\ A, i_2 = 0.25\ A, i_3 = 1.25\ A$$